

**University of Information Technology and Sciences**  
**Faculty of Science and Engineering**  
**Department of Computer Science and Engineering**  
**Program: B.Sc. in CSE**  
**Term Final Examination, Autumn-2022**  
**Course Title: Differential and Integral Calculus**  
**Course Code: MAT 163**

Marks: 50

Time: 3(three) hours

(Answer all questions)

1. a) Find the intervals on which the function  $f(x) = 3x^4 + 4x^3 - 12x^2 + 2$  is increasing and decreasing. [5]
- b) State Extreme-Value theorem. Find the absolute maximum and minimum values of the function  $f(x) = 2x^3 - 3x^2 - 12x$  on the interval  $[-3, 2]$ , and determine where these values occur. [5]
2. a) By using L'Hospital rule evaluate: [4]
- (i)  $\lim_{x \rightarrow 0} (1+x)^{1/x}$       (ii)  $\lim_{x \rightarrow 0} \left[ \frac{1}{x} - \frac{1}{\sin x} \right]$
- b) (i) If  $w = e^y \cos x$ , then evaluate the followings: [6]
- $\frac{\partial^3 w}{\partial y^2 \partial x} \Big|_{(\pi/4, 0)}$       and       $\frac{\partial^3 w}{\partial x^2 \partial y} \Big|_{(\pi/4, 0)}$
- (ii) Given  $f(x, y) = y^2 e^x + y$ . Find  $f_{xyy}$ .
3. a) Evaluate the following indefinite integrals: [5]
- (i)  $\int \frac{\sqrt{x^2-9}}{x} dx$       (ii)  $\int x^2 \sqrt{x+1} dx$
- b) Evaluate the following definite integrals: [5]
- (i)  $\int_0^{\ln 3} e^x (3 + e^x)^3 dx$
- (ii)  $\int_0^{\sqrt{3}/2} \sin^{-1} x dx$

4. a) Define Gamma and Beta function. Prove that  $\Gamma(n + 1) = n\Gamma(n)$ . [4]

b) By using gamma and beta function evaluate the following integrals. [6]

(i)  $\int_0^1 x^4(1 - x^2)^{3/2} dx$

(ii)  $\int_0^{\pi/2} \sin^5 \theta \cos^6 \theta d\theta$

(iii)  $\int_0^{\infty} e^{-x^2} dx$

5. a) Find the area of the region that is enclosed between the curves  $y = x^2$  and  $y = x + 6$ . [4]

b) Evaluate the following multiple integrals: [6]

(i)  $\int_0^1 \int_{-x}^{x^2} y^2 x dy dx$

(ii)  $\int_0^1 \int_0^y \int_0^{\sqrt{1-y^2}} z dz dy dx$